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Controllability of Fractional Nonlocal Quasilinear Evolution Inclusions with Resolvent Families

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Abstract

The aim of this paper is to establish a controllability result for a class of nonlocal quasilinear differential inclusions of fractional order in Banach spaces. We use the theory of fractional calculus, resolvent family and fixed point techniques for the main results. Furthermore, a suitable set of sufficient conditions is presented for the considered problem to be controllable. As an example that illustrates the abstract results, a fractional nonlocal control partial differential system is given.

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1 Introduction

Nowadays, fractional differential systems are applicable in various fields of science and engineering with more accurate and qualitative results, see the monographs of Baleanu et al. [5] and Malinowska & Torres [19]. The existence and controllability problems for dynamic systems by using the methods of fixed point theory have been investigated in many works, see for instance [13, 14, 21, 23]. In recent years, the study of differential inclusions and control problems have attracted the attention of many mathematicians and physicists [1, 2]. A large amount of articles developed concerning the investigation of controllability problems for systems governed by semilinear differential inclusions in

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Banach spaces [4, 6, 7, 11, 16]. However, there are few papers dealing with the controllability for systems governed by nonlocal quasilinear differential inclusions.

Motivated by Bragdi and Hazi [8], Górniewicz et al. [16] and Debbouche & Baleanu [12], our main purpose in this paper is to study the controllability of nonlocal quasilinear fractional differential inclusions by using a resolvent family, a fixed point theorem and multi-valued analysis [17]. We remark that the method used in this paper is different from that in [16].

The paper is organized as follows. In Section 2 we give the necessary preliminaries for the paper. In Section 3 we give the outline of the corresponding results for the case when the multivalued map is lower semicontinuous. The last two sections are devoted to an illustrated example and a remark.

2 Preliminaries

In this section, we give some basic definition and proprieties of fractional calculus and resolvent family.

The theory of fractional calculus is essentially based on the integral convolution between the function $x(t)$ and the following generalized function, introduced by Gelfand and Shilov [15], $\phi_\lambda(t) = t_+^\lambda / \Gamma(\lambda)$, where λ is a complex number, $t_+^\lambda = t^\lambda \theta(t)$, $\theta(t)$ being the Heaviside step function, and $\Gamma(\lambda)$ is the gamma function.

Following Gelfand and Shilov [15], we define the Riemann–Liouville fractional integral operator of order $\beta > 0$ by

$$I^\beta x(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} x(s) ds. \tag{2.1}$$

If $0 < \alpha \leq 1$, we can define the fractional derivative of order α in the Caputo sense as

$$\frac{d^\alpha x(t)}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x'(s)}{(t-s)^\alpha} ds, \tag{2.2}$$

where $x'(s) = \dfrac{dx(s)}{ds}$ (see also [18, 20, 22]).

If x is an abstract function with values in X, then the integrals and derivatives which appear in (2.1) and (2.2) are taken in Bochner's sense.

Now let X be a separable real Banach space with the norm $\|\cdot\|$. We denote by E the Banach space $C(J;X)$ of X-valued continuous functions on J equipped with the sup-norm and by B(X) the space of all bounded linear operators from X to X. For $x \in X$ and for nonempty sets A, B of X we denote $d(x, A) = \inf\{\|x-y\|; y \in B\}$, $e(A, B) := \sup\{d(x, B); x \in A\}$ and $\chi(A, B) := \max\{e(A, B), e(B, A)\}$ for a Hausdorff metric. For more details, we refer the reader to [10, 21].

Definition 2.1 (See [12])**.** Let $A(t, x)$ be a closed and linear operator with domain $D(A)$ defined on a Banach space X and $\alpha > 0$. Let $\rho(A(t, x))$ be the resolvent set of $A(t, x)$.

We call $A(t,x)$ the generator of an (α,x)-resolvent family if there exist $\omega \geq 0$ and a strongly continuous function $R_{(\alpha,x)} : \mathbb{R}_+^2 \to L(X)$ such that $\{\lambda^\alpha : \Re(\lambda) > \omega\} \subset \rho(A(t,x))$ and for $0 \leq s \leq t < \infty$,

$$[\lambda^\alpha I - A(s,x)]^{-1} x = \int_0^\infty e^{-\lambda(t-s)} R_{(\alpha,x)}(t,s) y dt, \;\; \Re(\lambda) > \omega, \;\; (x,y) \in X^2.$$

In this case, $R_{(\alpha,x)}(t,s)$ is called the (α,x)-resolvent family generated by $A(t,x)$.

Motivated by the work in [8, 12, 16], we consider the following control system governed by a fractional semilinear differential inclusion:

$$\frac{d^\alpha x(t)}{dt^\alpha} \in A(t,x(t))x(t) + F(t,x(t)) + Bu(t), \;\; t \in J := [0,b], \tag{2.3}$$

$$x(0) + h(x) = x_0, \tag{2.4}$$

where $0 < \alpha < 1$, $F : J \times X \to \mathcal{P}(X)$ is a bounded, closed, multivalued map, $\mathcal{P}(X)$ the set of all nonempty subsets of X, $x_0 \in X$ and $h : C(J,X) \to X$. We assume that $A(t,\cdot)$ is a closed linear operator defined on a dense domain $D(A)$ in X into X such that $D(A)$ is independent of t. It is assumed also that $A(t,\cdot)$ generates an (α,x)-resolvent family $R_{(\alpha,x)}(t,s)$ in the Banach space X and the control function $u(\cdot)$ is given in $L^2(J,U)$, a Banach space of admissible control functions with U as a Banach space. Finally B is a bounded linear operator from U to X. The following definition is similar to the concept defined in [3, 12, 16].

Definition 2.2. A continuous function $x(\cdot)$ is said to be a mild solution of the problem (2.3)–(2.4), if $x(0) + h(x) = x_0$ and there exists a function $v \in L^1(J,X)$ such that $v(t) \in F(t,x(t))$ a.e. on J and

$$x(t) = R_{(\alpha,x)}(t,0)x_0 - R_{(\alpha,x)}(t,0)h(x) + \int_0^t R_{(\alpha,x)}(t,s)\left[Bu(s) + v(s)\right] ds.$$

Definition 2.3 (See [12, 16]). The system (2.3)–(2.4) is said to be controllable on the interval J, if for every $x_0, \; x_1 \in X$ there exists a control $u \in L^2(J,U)$ such that the mild solution $x(t)$ of (2.3)–(2.4) satisfies $x(0) + h(x) = x_0$ and $x(b) = x_1$.

Also, we need the following lemma.

Lemma 2.4 (See [12, Lemma 3.1]). *Let $\Omega \subset X$, Y be a densely and continuously imbedded Banach space in X and let $R_{(\alpha,x)}(t,s)$ be the resolvent operator for the problem* (2.3)–(2.4). *Then there exists a constant $K > 0$ such that*

$$\|R_{(\alpha,y)}(t,s)\omega - R_{(\alpha,x)}(t,s)\omega\| \leq K\|\omega\|_Y \int_s^t \|y(\tau) - x(\tau)\| d\tau$$

for every z_1, $z_2 \in E$ with values in Ω and every $\omega \in Y$.

3 Main Results

To investigate the controllability of the system (2.3)–(2.4), we assume the following conditions:

(H1) The bounded linear operator $W : L^2(J, U) \to X$, defined by

$$Wu = \int_0^b R_{(\alpha,x)}(b, s)Bu(s)ds,$$

has an induced inverse operator $\tilde{W}^{-1}$ which takes values in $L^2(J, U)/\ker W$ and there exist positive constants M_1, M_2 such that $\|B\| \le M_1$ and $\| \tilde{W}^{-1} \| \le M_2$;

(H2) $F : J \times X \to \mathcal{P}(X)$ is a nonempty, compact-valued, multivalued map such that:

(i) $(t, x) \mapsto F(t, x)$ is $\mathcal{L} \otimes \mathcal{B}$ measurable,

(ii) $x \mapsto F(t, x)$ is lower semi-continuous for a.e. $t \in J$;

(H3) For each $k > 0$, there exists $\varphi_k(\cdot) \in L^1(J, \mathbb{R}_+)$ such that

$$\|F(t, x)\| = \sup\{\|v\| \mid v \in F(t, x)\} \le \varphi_k(t),$$

for all $\|x\| \le k$ and for a.e. $t \in J$;

(H4) There exists a function $\psi(\cdot) \in L^1(J, \mathbb{R}_+)$ such that F is $\psi(t)$-Lipschitz in the sense that

$$\chi(F(t, y), F(t, x)) \le \psi(t)\|y - x\|, \text{ for a.e. } t \in J,$$

and for each $x, y \in X$, where χ is the Hausdorff metric on $\mathcal{P}(X)$ (see [10, 21]);

(H5) $h : E \to Y$ is Lipschitz continuous in X and bounded in Y, that is, there exist positive constants H_1 and H_2 such that

$$\|h(x)\|_Y \le H_1,$$

$$\|h(y) - h(x)\|_Y \le H_2\|y - x\|_E, \quad x, y \in E;$$

(H6) $M = \max \|R_{(\alpha,x)}(t, s)\|_{B(X)}$, $0 \le s \le t \le b$, $x \in X$;

(H7) There exist positive constants k_1 and k_2 such that

$$M\|x_0\| + MM_1M_2\left[\|x_1\| + M\|x_0\| + MH_1 + M\int_0^b \varphi_{k_1}(s)\,ds\right] + M\|\varphi_{k_1}\|_{L^1} \le k_1$$

and

$$k_2 = MM_1M_2b\left[(Kb^2\|f\|_{L^1} + M\int_0^b \psi(s)ds) + (Kb\|x_0\| + MH_2 + KbH_1)\right]$$
$$+ KbM_1M_2\left[\|x_1\| + M\|x_0\| + MH_1 + M\int_0^b \varphi_{k_1}(s)\,ds\right] < 1.$$

Theorem 3.1. *If hypotheses* (H1)–(H7) *are satisfied, then system* (2.3)–(2.4) *is controllable on* J.

Proof. As in [16], the assumption (H2) and (H3) imply that F is lower semicontinuous (i.e., it has nonempty closed and decomposable values (see [9])), then there exists a continuous function $f : E \to L^1(J, X)$ such that $f(x) \in \mathcal{F}(x)$ for all $x \in E$, where $\mathcal{F}$ is the Nemitsky operator defined by

$$\mathcal{F}(x) = \left\{\, w \in L^1(J, X) \mid w(t) \in F(t, x(t)) \text{ for a.e. } t \in J \,\right\}.$$

Consider the problem

$$\frac{d^\alpha x(t)}{dt^\alpha} - A(t, x)x(t) - Bu(t) = f(x)(t), \quad t \in J, \tag{3.1}$$
$$x(0) + h(x) = x_0. \tag{3.2}$$

It is clear that if $x \in E$ is a solution of the problem (3.1)–(3.2), then it is also solution to the problem (2.3)–(2.4). In view of the hypothesis (H1) for an arbitrary function $x(t)$, the control is defined as follows (compare with [12])

$$u_x(t) = \tilde{W}^{-1}\left[x_1 - R_{(\alpha,x)}(b, 0)x_0 + R_{(\alpha,x)}(b, 0)h(x) - \int_0^b R_{(\alpha,x)}(b, s)f(x)(s)\,ds\right](t).$$

In what follows, it suffices to show that when using this control the operator $\Phi : E \to E$ defined by

$$\Phi x(t) = R_{(\alpha,x)}(t, 0)x_0 - R_{(\alpha,x)}(t, 0)h(x) + \int_0^t R_{(\alpha,x)}(t - s)\left[Bu_x(s) + f(x)(s)\right]ds,$$

$t \in J$, has a fixed point $x(\cdot)$ from which it follows that this fixed point is a mild solution of the system (2.1)–(2.2). Clearly $\Phi x(b) = x_1$, from which we conclude that the system is controllable.

Let B_{k_1} be the nonempty closed and bounded set given by

$$B_{k_1} = \{z \in E : z(0) = x_0,\ \|z\| \le k_1\}. \tag{3.3}$$

We first prove that $\Phi(B_{k_1}) \subset B_{k_1}$. For convenience we let

$$G_x(\eta) = B\tilde{W}^{-1}\left[x_1 - R_{(\alpha,x)}(b,0)x_0 + R_{(\alpha,x)}(b,0)h(x) - \int_0^b R_{(\alpha,x)}(b,s)f(x)(s)\,ds\right](\eta).$$

It is clear, for $x \in B_{k_1}$, that

$$\|G_x(\eta)\| \le M_1M_2\left[\|x_1\| + M\|x_0\| + MH_1 + M\int_0^b \varphi_{k_1}(s)\,ds\right] := G_0$$

and

$$\|\Phi x(t)\| \le M\|x_0\| + MG_0 + M\|\varphi_{k_1}\|_{L^1},$$

where the last two inequalities follow from (H1), (H3), (H5) and (H6).

From assumption (H7), one gets $\|\Phi x\| \le k_1$. Thus, Φ maps B_{k_1} into itself.

Now we prove that Φ is a contraction. For $x, y \in B_k$

$$\|\Phi y(t) - \Phi x(t)\| \le I_1(t) + I_2(t) + I_3(t),$$

where

$$I_1(t) = \|R_{(\alpha,x)}(t,0)x_0 - R_{(\alpha,y)}(t,0)x_0\| + \|R_{(\alpha,x)}(t,0)h(x) - R_{(\alpha,y)}(t,0)h(y)\|,$$

$$I_2(t) = \int_0^t \|R_{(\alpha,y)}(t,s)G_y(s) - R_{(\alpha,x)}(t,s)G_x(s)\|ds,$$

and

$$I_3(t) = \int_0^t \|R_{(\alpha,y)}(t,s)f(y)(s) - R_{(\alpha,x)}(t,s)f(x)(s)\|ds.$$

Applying Lemma 2.4, (H5) and (H6), we get

$$\begin{aligned}I_1(t) &\le \|R_{(\alpha,x)}(t,0)x_0 - R_{(\alpha,y)}(t,0)x_0\| + \|R_{(\alpha,x)}(t,0)h(x) - R_{(\alpha,x)}(t,0)h(y)\| \\ &\quad + \|R_{(\alpha,x)}(t,0)h(y) - R_{(\alpha,y)}(t,0)h(y)\| \\ &\le (Kb\|x_0\| + MH_2 + KbH_1)\|x-y\|_E.\end{aligned}$$

From Lemma 2.4 and (H4)–(H6), we have

$$\begin{aligned}
I_2(t) &\leq \int_0^t [\|R_{(\alpha,y)}(t,s)G_y(s) - R_{(\alpha,y)}(t,s)G_x(s)\| \\
&\quad + \|R_{(\alpha,y)}(t,s)G_x(s) - R_{(\alpha,x)}(t,s)G_x(s)\|]ds \\
&\leq \int_0^t [M\|G_y(s) - G_x(s)\| + Kb\|G_x(s)\|\|y-x\|_E]ds \\
&\leq \int_0^t [MM_1M_2(I_1(b) + I_3(b)) + KbG_0\|y-x\|_E]ds \\
&\leq \left\{ MM_1M_2b[(Kb^2\|f\|_{L^1} + M\int_0^b \psi(s)ds) + (Kb\|x_0\| + MH_2 + KbH_1)] \right. \\
&\quad \left. + KbG_0 \right\} \|y-x\|_E.
\end{aligned}$$

Again, we apply Lemma 2.4, and (H4)–(H6), to obtain

$$\begin{aligned}
I_3(t) &\leq \int_0^t [\|R_{(\alpha,y)}(t,s)f(y)(s) - R_{(\alpha,y)}(t,s)f(x)(s)\| \\
&\quad + \|R_{(\alpha,y)}(t,s)f(x)(s) - R_{(\alpha,x)}(t,s)f(x)(s)\|]ds \\
&\leq \int_0^t M\,[\|f(y)(s) - f(x)(s)\| + Kb\|f\|_{L^1}]\,ds \\
&\leq \left(Kb^2\|f\|_{L^1} + M\int_0^b \psi(s)ds \right) \|y-x\|_E.
\end{aligned}$$

Now, from (H5), we have

$$\|\Phi y\,(t) - \Phi x\,(t)\| \leq k_2\|y-x\|_E.$$

Therefore, Φ is a contraction mapping and hence there exists a unique fixed point $x \in X$, which is a solution to problem 3.1–3.2. This completes the proof. □

In next section we present an example to illustrate our main results.

4 Example

Consider the following partial control differential system:

$$\frac{\partial^\alpha w(y,t)}{\partial t^\alpha} = a(y,t,w(y,t))\frac{\partial^2 w(y,t)}{\partial y^2} + \mathcal{Q}\,(t, w\,(y,t)) + \mu\,(y,t)\,, \tag{4.1}$$

$$w(y,0) + \sum_{i=1}^{p} c_i w(y,t_i) = w_0(y),\ y \in [0,\pi], \tag{4.2}$$

$$w(0,t) = w(\pi,t) = 0,\ t \in J := [0,b], \tag{4.3}$$

where $0 < \alpha < 1,\ 0 < t_1 < t_2 < \cdots < t_p \leq b,\ c_i \in \mathbb{R}$, $a : (0, \pi) \times [0, b] \times \mathbb{R} \to \mathbb{R}$, $\mu : [0, \pi] \times J \to [0, \pi]$ is continuous and $\mathcal{Q} : J \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a multivalued map.

Let us take $X = U = L^2 [0, \pi]$, $u(t) = \mu(t, \cdot)$, $x(t) = w(t, \cdot)$, $h(w(\cdot, t)) = \sum_{i=1}^{p} c_k w(\cdot, t_i)$ and

$$F(t, x(t))(y) = \mathcal{Q}(t, w(t, y)), \quad t \in J,\ y \in [0, \pi].$$

such that the hypotheses (H2)–(H4) are satisfied. We define $A(t, \cdot) : X \to X$ by $(A(t, \cdot)z)(y) = a(y, t, \cdot)z''$ with

(i) The domain $D(A(t, \cdot)) = \{z \in X : z, z'$ are absolutely continuous, $z'' \in X$ and $z(0) = z(\pi) = 0\}$ dense in the Banach space X and independent of t. Then A can be written as

$$A(t, x)z = -\sum_{n=1}^{\infty} n^2 (z, z_n) z_n,\ z \in D(A),$$

where $(\cdot, \cdot)$ is the inner product in $L^2[0, \pi]$ and $z_n = Z_n \circ x$ is the orthogonal set of eigenvectors in $A(t, x)$, and $Z_n(t, s) = \sqrt{2/\pi} \sin n(t - s)^\alpha$, $0 \leq s \leq t \leq b$, $n = 1, 2, \ldots$;

(ii) The operator $[A(t, \cdot) + \lambda^\alpha I]^{-1}$ exists in $L(X)$ for any λ with $\Re(\lambda) \leq 0$ and

$$\|[A(t, \cdot) + \lambda^\alpha I]^{-1}\| \leq \frac{C_\alpha}{|\lambda| + 1},\ t \in J;$$

(iii) There exist constants $\eta \in (0, 1]$ and C_α such that

$$\|[A(t_1, \cdot) - A(t_2, \cdot)]A^{-1}(s, \cdot)\| \leq C_\alpha |t_1 - t_2|^\eta,\ \ t_1, t_2, s \in J.$$

Under these conditions each operator $A(s, \cdot)$, $s \in J$, generates an evolution operator, which is an (α, x)-resolvent family and has the form (see [12]):

$$R_{\alpha,x}(t, s)z = \sum_{n=1}^{\infty} \exp[-n^2(t - s)^\alpha](z, z_n)z_n,\ z \in X.$$

With this choice of A, F and $(Bu)(y, t) = \mu(t, y)$, we see that (2.3)–(2.4) are the abstract formulation of (4.1)–(4.2). Now assume that the linear operator W given by

$$Wu(y) = \sum_{n=1}^{\infty} \int_0^b \exp[-n^2(b - t)^\alpha](\mu(y, s), z_n)z_n ds,\ \ y \in [0, \pi]$$

has a bounded invertible operator $\tilde{W}$ in $L^2(J, U)/\ker W$. Thus all the conditions of the Theorem 3.1 are satisfied. Hence system (4.1)–(4.2) is controllable on J.

5 Remark

The controllability for semilinear fractional order systems, when the nonlinear term is independent of the control function, is proved by assuming that the controllability operator has an induced inverse on a quotient space, see for example [12, 13]. However, if the semigroup associated with the system is compact, then the controllability operator is also compact and hence the induced inverse does not exist because the state space is infinite dimensional [24]. Thus, the concept of exact controllability is too strong, i.e., the results cannot be applicable. In this case the notion of approximate controllability is more appropriate for these control systems. We can define the approximate controllability for the control system (2.3)–(2.4) as follows. Let $x_b(x_0 - h(x); u)$ be the state value of (2.3)–(2.4) at terminal time b corresponding to the control u and the nonlocal value $x(0)$. For every $x_0 \in X$, we introduce the set

$$\mathfrak{R}(b, x(0)) = \left\{ x_b\left(x_0 - h(x(t)); u\right)(0) : u(\cdot) \in L^2(J, U) \right\},$$

which is called the reachable set of system (2.3)–(2.4) at terminal time b (with U a Banach space). Its closure in X is denoted by $\overline{\mathfrak{R}(b, x(0))}$.

Definition 5.1. The system (2.3)–(2.4) is said to be approximately controllable on J if $\overline{\mathfrak{R}(b, x(0))} = X$, that is, given an arbitrary $\epsilon > 0$, it is possible to steer the system from the point $x(0)$ at time b to all points in the state space X within a distance ϵ.

Consider the following linear control fractional system

$$D_t^\alpha x(t) = A(t, x(t))x(t) + Bu(t), \tag{5.1}$$

$$x(0) = x_0. \tag{5.2}$$

The approximate controllability for the fractional linear control system (5.1)–(5.2) is a natural generalization of the notion of approximate controllability of a first-order linear control system. It is convenient at this point to introduce the controllability operators associated with (5.1)–(5.2) as

$$\Gamma_0^b = \int_0^b R_{(\alpha,x)}(b, s)BB^* R^*_{(\alpha,x)}(b, s)ds,$$

where B^* and $R^*_{(\alpha,x)}$ denote the adjoints of B and $R_{(\alpha,x)}$, respectively. Moreover, we give the relevant operator

$$\mathcal{R}(\lambda, \Gamma_0^b) = (\lambda I + \Gamma_0^b)^{-1},$$

for $\lambda > 0$. It is straightforward to see that the operator Γ_0^b is a linear bounded operator.

Lemma 5.2 (See [25]) The linear fractional control system (5.1)–(5.2) is approximately controllable on J if and only if $\lambda\mathcal{R}(\lambda, \Gamma_0^b) \to 0$ as $\lambda \to 0^+$, in the strong operator topology.

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