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Existence of Positive Periodic Solutions of Mutualism Systems with Several Delays[1]

Yonghui Xia
College of Mathematics and Computer Science,
Fuzhou University, Fuzhou 350002, P.R.China
E-mail: yhxia@fzu.edu.cn; xiayonhui@163.com

Abstract

In this paper, by using Mawhin coincidence degree, we study the global existence of positive periodic solutions for a class of mutualism systems with several delays and obtain a new and interesting criterion, which is much different from those of Yang etc. [8]. Moreover, our arguments for obtaining bounds of solutions to the operator equation $Lx = \lambda Nx$ are much different from those used in [8]. So some new arguments are employed for the first time.

AMS subject classification: 34C25, 92D25, 92B05.
Keywords: Periodic solutions, coincidence degree, mutualism model.

1. Introduction

Traditional Lotka–Volterra type predator-prey model or competitive model has received great attention from both theoretical and mathematical biologists and has been studied extensively (for example, see [1–5, 9, 11–22]). But there are few papers considering the mutualism system. Goh [6] discussed the stability in models of mutualism of the form

$$\begin{cases} \dot{x}_1(t) &= x_1(t)\big[r_1(t) - a_{11}(t)x_1(t) + a_{12}(t)x_2(t)\big], \\ \dot{x}_2(t) &= x_2(t)\big[r_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\big]. \end{cases} \tag{1.1}$$

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Then Gopalsamy and He [7] studied the following system with discrete delay

$$\begin{cases} \dot{x}_1(t) &= x_1(t)\big[r_1(t) - a_{11}(t)x_1(t-\tau) + a_{12}(t)x_2(t-\tau)\big], \\ \dot{x}_2(t) &= x_2(t)\big[r_2(t) + a_{21}(t)x_1(t-\tau) - a_{22}(t)x_2(t-\tau)\big]. \end{cases} \tag{1.2}$$

Some sufficient conditions were obtained for the persistence and global attractivity of system (1.2). Then Yang etc. [8] obtained some sufficient conditions for the existence of positive periodic solutions of the system

$$\begin{cases} \dot{y}_1(t) &= y_1(t)\big[r_1(t) - a_{11}(t)y_1(t-\tau_1(t)) + a_{12}(t)y_2(t-\tau_2(t))\big], \\ \dot{y}_2(t) &= y_2(t)\big[r_2(t) + a_{21}(t)y_1(t-\tau_1(t)) - a_{22}(t)y_2(t-\tau_2(t))\big]. \end{cases} \tag{1.3}$$

Now we generalize the system to

$$\begin{cases} \dot{y}_1(t) &= y_1(t)\big[r_1(t) - a_{11}(t)y_1(t-\tau_1(t)) + a_{12}(t)y_2(t-\tau_2(t))\big], \\ \dot{y}_2(t) &= y_2(t)\big[r_2(t) + a_{21}(t)y_1(t-\sigma_1(t)) - a_{22}(t)y_2(t-\sigma_2(t))\big], \end{cases} \tag{1.4}$$

where $a_{ij}(t), i, j = 1, 2, \tau_i(t), \sigma_i(t), i = 1, \dots, n$ are positive periodic functions with period $\omega > 0$ and $\displaystyle\int_0^\omega r_i(t)\mathrm{d}t > 0, i = 1, 2$. The assumption of periodicity of the parameters $a_{ij}(t)$ is a way of incorporating the periodicity of the environment (e.g., seasonal effects of weather condition, food supplies, temperature, mating habits, harvesting etc.). The growth functions $r_i(t)$ are unnecessary to remain positive, since the environment fluctuates randomly, in bad condition, $r_i(t)$ may be negative.

Obviously, system (1.3) is a special case of system (1.4). Unlike system (1.3), $\sigma_i(t) \neq \tau_i(t)$. Thus the existing argument in Yang etc. [8] for obtaining bounds of solutions to the operator equation $Lx = \lambda Nx$ are not applicable to our case and some new arguments are employed for the first time.

The initial conditions for system (1.4) take the form of

$$\begin{cases} \dot{y}_i(s) = \psi_i(s), \quad \tau = \max \sup\limits_{t\in[0,\omega]} \{\tau_i(t), \sigma_i(t), i = 1, 2\} \\ \dot{\psi}_i(t) \in C([-\tau, 0], R_+), \psi_i(0) > 0, \ i = 1, 2. \end{cases} \tag{1.5}$$

Throughout this paper, we will use the following natation:

$$\overline{f} = \frac{1}{\omega}\int_0^\omega f(t)\mathrm{d}t, \ f^\iota = \min_{t\in[0,\omega]} f(t), \ f^\mu = \max_{t\in[0,\omega]} f(t),$$

where $f(t)$ is an ω-periodic function.

2. Existence of Positive Periodic Solutions

In order to obtain the existence of positive periodic solutions of (1.4), for convenience, we shall summarize in the following a few concepts and results from [10] that will be prerequisite for this section.

Let X, Y be real Banach spaces, let $L : \mathrm{Dom}L \subset X \to Y$ be a linear mapping, and $N : X \to Y$ be a continuous mapping. The mapping L is called a Fredholm mapping of index zero if dimKer$L = \mathrm{codim}\,\mathrm{Im}L < +\infty$ and ImL is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \to X$, and $Q : Y \to Y$ such that Im$P = \mathrm{Ker}L$, $\mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I - Q)$. It follows that $L|\mathrm{dom}L \cap \mathrm{Ker}P : (I - P)X \to \mathrm{Im}L$ is invertible. We denote the inverse of that map by K_p. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_p(I - Q)N : \overline{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 2.1. [10] Let $\Omega \subset X$ be an open bounded set. Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Assume

(a) for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \mathrm{Dom}L$, $Lx \neq \lambda Nx$;

(b) for each $x \in \partial\Omega \cap \mathrm{Ker}L$, $QNx \neq 0$;

(c) $\deg\{JQN,\ \Omega \cap \mathrm{Ker}L, 0\} \neq 0$.

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \mathrm{Dom}L$.

Lemma 2.2. Assume that f, g are continuous nonnegative functions defined on the interval $[a, b]$. Then there exists $\xi \in [a, b]$ such that

$$\int_a^b f(t)g(t)\mathrm{d}t = f(\xi)\int_a^b g(t)\mathrm{d}t.$$

Theorem 2.3. Let

$$b_{11}^{\iota} = \min_{t\in[0,\omega]} \frac{a_{11}(t)}{1 - \dot{\tau}_1(t)}, \quad b_{12}^{\mu} = \max_{t\in[0,\omega]} \frac{a_{12}(t)}{1 - \dot{\tau}_2(t)},$$

$$b_{22}^{\iota} = \min_{t\in[0,\omega]} \frac{a_{22}(t)}{1 - \dot{\sigma}_2(t)}, \quad b_{21}^{\mu} = \max_{t\in[0,\omega]} \frac{a_{21}(t)}{1 - \dot{\sigma}_1(t)}.$$

If $b_{11}^{\iota}b_{22}^{\iota} > b_{21}^{\mu}b_{12}^{\mu}$ and $\dot{\tau}_i(t) < 1$, $\dot{\sigma}_i(t) < 1$, $(i = 1, 2)$ hold, then system (1.4) has at least one positive ω-periodic solution.

Proof. We make the change of variables

$$x_i(t) = \ln y_i(t), \quad i = 1, 2. \tag{2.1}$$

Then (1.4) is rewritten as

$$\left\{\begin{array}{lcl} \dot{x}_1(t) & = & r_1(t) - a_{11}(t)\exp\{x_1(t - \tau_1(t))\} + a_{12}(t)\exp\{x_2(t - \tau_2(t))\}, \\ \dot{x}_2(t) & = & r_2(t) + a_{21}(t)\exp\{x_1(t - \sigma_1(t))\} - a_{22}(t)\exp\{x_2(t - \sigma_2(t))\}. \end{array}\right. \tag{2.2}$$

Take

$$X = Y = \{x = (x_1, x_2)^T \in C(\mathbb{R}, \mathbb{R}^2) : x(t+\omega) = x(t)\}$$

and define

$$\|x\| = \max_{t\in[0,\omega]} |x_1(t)| + \max_{t\in[0,\omega]} |x_2(t)|, \quad x = (x_1, x_2)^T \in X \text{ or } Y$$

(here $|\cdot|$ denotes the Euclidean norm). Then X and Y are Banach spaces with the norm $\|\cdot\|$. For any $x = (x_1, x_2)^T \in X$, because of the periodicity, we can easily check that

$$r_1(t) - a_{11}(t)\exp\{x_1(t-\tau_1(t))\} + a_{12}(t)\exp\{x_2(t-\tau_2(t))\} := \Delta_1(x,t) \in C(\mathbb{R}, \mathbb{R}),$$

$$r_2(t) + a_{21}(t)\exp\{x_1(t-\sigma_1(t))\} - a_{22}(t)\exp\{x_2(t-\sigma_2(t))\} := \Delta_2(x,t) \in C(\mathbb{R}, \mathbb{R}),$$

are ω-periodic. Set

$$L : \mathrm{Dom}L \cap X, \ \ L(x_1(t), x_2(t)) = \Big(\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}, \frac{\mathrm{d}x_2(t)}{\mathrm{d}t}\Big),$$

where $\mathrm{Dom}L = \{(x_1(t), x_2(t)) \in C^1(R, R^2)\}$ and $N : X \to X$,

$$N\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \Delta_1(x,t) \\ \Delta_2(x,t) \end{pmatrix}.$$

Define

$$P\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Q\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\omega}\displaystyle\int_0^\omega x_1(t)\mathrm{d}t \\ \dfrac{1}{\omega}\displaystyle\int_0^\omega x_2(t)\mathrm{d}t \end{pmatrix}, \ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in X = Y.$$

It is not difficult to show that

$$\mathrm{Ker}L = \{x | x \in X, \ x = C_0, \ C_0 \in \mathbb{R}^2\},$$
$$\mathrm{Im}L = \Big\{y | y \in Y, \ \int_0^\omega y(t)\mathrm{d}t = 0\Big\} \text{ is closed in } Y,$$
$$\dim \mathrm{Ker}L = \mathrm{codim}\ \mathrm{Im}L = 2,$$

and P and Q are continuous projections such that

$$\mathrm{Im}P = \mathrm{Ker}L, \quad \mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I-Q).$$

It follows that L is a Fredholm mapping of index zero. Furthermore, the inverse K_p of L_p exists and has the form $K_p : \mathrm{Im}L \to \mathrm{Dom}L \cap \mathrm{Ker}P$

$$K_p(y) = \int_0^t y(s)\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^t y(s)\mathrm{d}s\mathrm{d}t.$$

Then $QN : X \to Y$ and $K_p(I-Q)N : X \to X$ read

$$QNx = \begin{pmatrix} \dfrac{1}{\omega}\displaystyle\int_0^\omega \Delta_1(x,t)\mathrm{d}t \\ \dfrac{1}{\omega}\displaystyle\int_0^\omega \Delta_2(x,t)\mathrm{d}t \end{pmatrix}$$

$$K_p(I-Q)Nx = \int_0^t Nx(s)\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^t Nx(s)\mathrm{d}s\mathrm{d}t - \Big(\frac{t}{\omega} - \frac{1}{2}\Big)\int_0^\omega Nx(s)\mathrm{d}s.$$

Clearly, QN and $K_p(I-Q)N$ are continuous. By using the Arzelà–Ascoli Theorem, it is not difficult to prove that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$.

Now we reach the position to search for an appropriate open bounded subset Ω for the application of Lemma 2.1.

Corresponding to the operator equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, we have

$$\begin{cases} \dot{x}_1(t) = \lambda\big[r_1(t) - a_{11}(t)\exp\{x_1(t-\tau_1(t))\} + a_{12}(t)\exp\{x_2(t-\tau_2(t))\}\big], \\ \dot{x}_2(t) = \lambda\big[r_2(t) + a_{21}(t)\exp\{x_1(t-\sigma_1(t))\} - a_{22}(t)\exp\{x_2(t-\sigma_2(t))\}\big]. \end{cases} \tag{2.3}$$

Suppose $x = (x_1, x_2)^T \in X$ is a solution of (2.3) for a certain $\lambda \in (0,1)$. Integrating (2.3) over the interval $[0,\omega]$, we obtain

$$\begin{cases} \displaystyle\int_0^\omega \big[r_1(t) - a_{11}(t)\exp\{x_1(t-\tau_1(t))\} + a_{12}(t)\exp\{x_2(t-\tau_2(t))\}\big]\mathrm{d}t = 0, \\ \displaystyle\int_0^\omega \big[r_2(t) + a_{21}(t)\exp\{x_1(t-\sigma_1(t))\} - a_{22}(t)\exp\{x_2(t-\sigma_2(t))\}\big]\mathrm{d}t = 0. \end{cases}$$

Hence,

$$\bar{r}_1\omega + \int_0^\omega a_{12}(t)\exp\{x_2(t-\tau_2(t))\}\mathrm{d}t = \int_0^\omega a_{11}(t)\exp\{x_1(t-\tau_1(t))\}\mathrm{d}t, \tag{2.4}$$

$$\bar{r}_2\omega + \int_0^\omega a_{21}(t)\exp\{x_1(t-\sigma_1(t))\}\mathrm{d}t = \int_0^\omega a_{22}(t)\exp\{x_2(t-\sigma_2(t))\}\mathrm{d}t. \tag{2.5}$$

Let $s = t - \tau_1(t)$. We note that $\dot{\tau}_1(t) < 1$, which implies $\dfrac{\mathrm{d}s}{\mathrm{d}t} = 1 - \dot{\tau}_1(t) > 0$, $\mathrm{d}s = (1-\dot{\tau}_1(t))\mathrm{d}t$. Therefore, the function $s = t - \tau_1(t)$ has the continuous inverse function $t = \tau_1^*(s)$, $s \in [-\tau_1(0), \omega - \tau_1(\omega)]$. So we have

$$\int_0^\omega a_{11}(t)\exp\{x_1(t-\tau_1(t))\}\mathrm{d}t = \int_{-\tau_1(0)}^{\omega-\tau_1(\omega)} \frac{a_{11}(\tau_1^*(s))}{1-\dot{\tau}_1(\tau_1^*(s))}\exp\{x_1(s)\}\mathrm{d}s.$$

By Lemma 2.2, there exists $\xi \in [-\tau_1(0), \omega - \tau_1(\omega)] = [-\tau_1(0), \omega - \tau_1(0)]$ such that

$$\begin{aligned}\int_0^{\omega} a_{11}(t)\exp\{x_1(t-\tau_1(t))\}\mathrm{d}t &= \frac{a_{11}(\tau_1^*(\xi))}{1-\dot{\tau}_1(\tau_1^*(\xi))}\int_{-\tau_1(0)}^{\omega-\tau_1(\omega)}\exp\{x_1(s)\}\mathrm{d}s \\ &= \frac{a_{11}(\eta_1)}{1-\dot{\tau}_1(\eta_1)}\int_0^{\omega}\exp\{x_1(t)\}\mathrm{d}t,\ (\eta_1 = \tau_1^*(\xi)).\end{aligned}$$

For the equations (2.3) and (2.4), similar to the above discussion, there exist $\eta_i \in [0, \omega], (i = 1, 2, 3, 4)$ such that

$$\bar{r}_1\omega + \frac{a_{12}(\eta_2)}{1-\dot{\tau}_2(\eta_2)}\int_0^{\omega}\exp\{x_2(t)\}\mathrm{d}t = \frac{a_{11}(\eta_1)}{1-\dot{\tau}_1(\eta_1)}\int_0^{\omega}\exp\{x_1(t)\}\mathrm{d}t, \tag{2.6}$$

$$\bar{r}_2\omega + \frac{a_{21}(\eta_3)}{1-\dot{\sigma}_1(\eta_3)}\int_0^{\omega}\exp\{x_1(t)\}\mathrm{d}t = \frac{a_{22}(\eta_4)}{1-\dot{\sigma}_2(\eta_4)}\int_0^{\omega}\exp\{x_2(t)\}\mathrm{d}t. \tag{2.7}$$

We denote

$$b_{11} = \frac{a_{11}(\eta_1)}{1-\dot{\tau}_1(\eta_1)},\quad b_{12} = \frac{a_{12}(\eta_2)}{1-\dot{\tau}_2(\eta_2)},\quad b_{21} = \frac{a_{21}(\eta_2)}{1-\dot{\sigma}_1(\eta_2)},\quad b_{22} = \frac{a_{22}(\eta_4)}{1-\dot{\sigma}_2(\eta_4)}.$$

It follows from (2.6)–(2.7) that

$$\bar{r}_1\omega + b_{12}\int_0^{\omega}\exp\{x_2(t)\}\mathrm{d}t = b_{11}\int_0^{\omega}\exp\{x_1(t)\}\mathrm{d}t, \tag{2.8}$$

$$\bar{r}_2\omega + b_{21}\int_0^{\omega}\exp\{x_1(t)\}\mathrm{d}t = b_{22}\int_0^{\omega}\exp\{x_2(t)\}\mathrm{d}t. \tag{2.9}$$

Under the assumption of Theorem 2.3, it is not difficult to derive that

$$\int_0^{\omega}\exp\{x_1(t)\}\mathrm{d}t = \frac{b_{12}\bar{r}_2 + b_{22}\bar{r}_1}{b_{11}b_{22} - b_{21}b_{12}}\omega \le \frac{b_{12}^{\mu}\bar{r}_2 + b_{22}^{\mu}\bar{r}_1}{b_{11}^{\iota}b_{22}^{\iota} - b_{21}^{\mu}b_{12}^{\mu}}\omega \tag{2.10}$$

and

$$\int_0^{\omega}\exp\{x_2(t)\}\mathrm{d}t = \frac{b_{21}\bar{r}_1 + b_{11}\bar{r}_2}{b_{11}b_{22} - b_{21}b_{12}}\omega \le \frac{b_{21}^{\mu}\bar{r}_1 + b_{11}^{\mu}\bar{r}_2}{b_{11}^{\iota}b_{22}^{\iota} - b_{21}^{\mu}b_{12}^{\mu}}\omega. \tag{2.11}$$

Therefore, there exists $t_i \in [0, \omega], i = 1, 2$ such that

$$x_1(t_1) \le \ln\frac{b_{12}^{\mu}\bar{r}_2 + b_{22}^{\mu}\bar{r}_1}{b_{11}^{\iota}b_{22}^{\iota} - b_{21}^{\mu}b_{12}^{\mu}},\quad x_2(t_2) \le \ln\frac{b_{21}^{\mu}\bar{r}_1 + b_{11}^{\mu}\bar{r}_2}{b_{11}^{\iota}b_{22}^{\iota} - b_{21}^{\mu}b_{12}^{\mu}}.$$

Obviously, there exists positive constants $M_i > 0, i = 1, 2$ such that

$$|x_i(t_i)| \le M_i,\ \ i = 1, 2. \tag{2.12}$$

On the other hand, it follows from (2.3), (2.4) and (2.10) that

$$\begin{aligned}\int_0^\omega |\dot{x}_1(t)|dt &= \lambda \int_0^\omega \Big| r_1(t) - a_{11}(t)\exp\{x_1(t-\tau_1(t))\} \\ &\quad + a_{12}(t)\exp\{x_2(t-\tau_2(t))\}\Big| \mathrm{d}t, \\ &< \bar{r}_1\omega + \int_0^\omega a_{12}(t)\exp\{x_2(t-\tau_2(t))\}\mathrm{d}t \\ &\quad + \int_0^\omega a_{11}(t)\exp\{x_1(t-\tau_1(t))\}\mathrm{d}t \\ &= 2\int_0^\omega a_{11}(t)\exp\{x_1(t-\tau_1(t))\}\mathrm{d}t \\ &\le 2b_{11}^\mu \int_0^\omega \exp\{x_1(t)\}\mathrm{d}t \le 2b_{11}^\mu \frac{b_{12}^\mu \bar{r}_2 + b_{22}^\mu \bar{r}_1}{b_{11}^\iota b_{22}^\iota - b_{21}^\mu b_{12}^\mu}\omega := H_1.\end{aligned} \tag{2.13}$$

Similarly, it follows from (2.3), (2.5) and (2.11) that

$$\int_0^\omega |\dot{x}_2(t)|\mathrm{d}t < 2b_{11}^\mu \frac{b_{21}^\mu \bar{r}_1 + b_{11}^\mu \bar{r}_2}{b_{11}^\iota b_{22}^\iota - b_{21}^\mu b_{12}^\mu}\omega := H_2. \tag{2.14}$$

It follows from (2.12)–(2.14) that

$$|x_i(t)| \le |x_i(t_i)| + \int_0^\omega |\dot{x}_i(t)|\mathrm{d}t < M_i + H_i.$$

Clearly, $M_i, H_i, i = 1, 2$ are independent of the choice of λ. We note that

$$\begin{aligned}\bar{a}_{11} &= \frac{1}{\omega}\int_0^\omega a_{11}(t)dt = \frac{1}{\omega}\int_{-\tau_1(0)}^{\omega-\tau_1(0)} \frac{a_{11}(\tau_1^*(s))}{1-\dot{\tau}_1(\tau_1^*(s))}\mathrm{d}s \\ &= \frac{a_{11}(\tau_1^*(s_1))}{1-\dot{\tau}_1(\tau_1^*(s_1))}, \quad s_1 \in [-\tau_1(0), \omega-\tau_1(0)],\end{aligned} \tag{2.15}$$

where $t = \tau_1^*(s)$ is the inverse function of $s = t - \tau_1(t)$. Similar to the discussion of (2.15),

$$\bar{a}_{12} = \frac{a_{12}(\tau_2^*(s_2))}{1-\dot{\tau}_2(\tau_2^*(s_2))}, \quad \bar{a}_{21} = \frac{a_{21}(\sigma_1^*(s_3))}{1-\dot{\sigma}_1(\sigma_1^*(s_3))}, \quad \bar{a}_{22} = \frac{a_{22}(\sigma_2^*(s_4))}{1-\dot{\sigma}_2(\sigma_2^*(s_4))}. \tag{2.16}$$

Thus, under the assumptions in Theorem 2.3, it follows from (2.15)–(2.16) that

$$\bar{a}_{11}\bar{a}_{22} - \bar{a}_{12}\bar{a}_{21} > b_{11}^\iota b_{22}^\iota - b_{12}^\mu b_{21}^\mu > 0. \tag{2.17}$$

Now it is easy to show that the system of algebraic equations

$$\bar{r}_i - \bar{a}_{ii}v_i + \bar{a}_{ij}v_j = 0, \;\; i, j = 1, 2.$$

has a unique solution $(v_1^*, v_2^*)^T \in R_+^2$ with $x_i^* > 0$. Take $H = \max\{M_i + H_i + C,\ M_i + H_i + C\}$, where $C > 0$ is taken sufficiently large such that $\|(\ln v_1^*, \ln v_2^*)^T\| < C$. Define $\Omega = \{x(t) \in X : \|x\| < H\}$. It is clear that Ω satisfies condition (a) of Lemma 2.1. Let $x \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap \mathbb{R}^2$, x be a constant vector in $\mathbb{R}^2$ with $\|x\| = H$. Then

$$QNx = \left[\ \bar{r}_i - \bar{a}_{ii} \exp\{x_i\} + \bar{a}_{ij} \exp\{x_j\} = 0\ \right]_{2\times 1} \neq 0.$$

Furthermore, let $J : \mathrm{Im}Q \to \mathrm{Ker}L$. In view of assumptions in Theorem 2.3 and (2.15)–(2.17), it is easy to see that

$$\deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} = \mathrm{sgn}\det \begin{bmatrix} -\bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & -\bar{a}_{22} \end{bmatrix} \geq \mathrm{sgn}(b_{11}^{\iota} b_{22}^{\iota} - b_{12}^{\mu} b_{21}^{\mu}) \neq 0,$$

where $\deg(\cdot)$ is the Brouwer degree and the J is the identity mapping since $\mathrm{Im}Q = \mathrm{Ker}L$.

By now, we have shown that Ω verifies all requirements of Lemma 2.1. Then it follows that $Lx = Nx$ has at least one solution in $\mathrm{Dom}L \cap \overline{\Omega}$. By (2.1), we derive that (1.4) has at least one positive ω-periodic solution. The proof is completed. ■

Remark 2.4. Our results and the method used in the proof are much different from those in the known literature. We remark that we generalize system (1.3) in [8] to system (1.4), where $\tau_i(t) \neq \sigma_i(t)$, i.e., all the delays are different. It is in this aspect that we generalize the results in [8].

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